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# Bifurcation Analysis of a Series Compensated System with STATCOM-ES

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**Abstract**

The energy storage devices incorporated into FACTS controllers increase the operational flexibility and reduce capital cost. This paper investigates the effect of Static Synchronous Compensator with energy storage (STATCOM-ES) on dynamic bifurcations of Subsynchronous resonance (SSR). The test system considered is IEEE first benchmark model (FBM).The line is provided with fixed series compensation and active shunt compensation at the electrical center of the line. The degree of series compensation is taken as the bifurcation parameter. The results of bifurcation analysis are validated by performing transient simulation of the system.



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*Keywords:*Bifurcation, Energy storage, FACTS, Hopf bifurcation, SSR, STATCOM

## 1. Introduction

The integration of energy storage devices with FACTS controllers to improve stability and power quality is a major challenge in a smart grid environment. A long transmission line requires series as well as shunt compensation to enhance power transfer capability and voltage regulation. But series compensation sometimes causes subsynchronous resonance problem, when the line is connected to turbo-alternators. This may lead to adverse torsional interaction [1-2]. In this paper, a series compensated system incorporating STATCOM with energy storage is considered for the analysis of subsynchronous resonance.

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STATCOM is a voltage source converter based shunt FACTS controller which regulates voltage at critical buses and improve system stability. It offers one degree of freedom in the absence of energy storage device and hence exchanges reactive power with the power system [3]. When an energy storage device is connected to the dc side of STATCOM, then it has two degrees of freedom (real and reactive current control). Therefore both active and reactive power can be exchanged with the system. It has been shown in [4-5] that the stability of the power system can be enhanced when STATCOM is integrated with energy storage devices. In [6], an adaptive controller is designed for a STATCOM with energy storage to damp power oscillations in a system.

Linear and nonlinear analysis of SSR, when the line is compensated with various FACTS controllers are reported in [7-8].Bifurcation analysis is a nonlinear analysis which provides an explanation for various nonlinear behaviors exhibited by a dynamical system. In [9-10], bifurcation technique is used to explain the system behavior which is experiencing SSR. In this paper, the impact of STATCOM-ES on a power system experiencing SSR is analyzed using bifurcation technique. The IEEE FBM is considered for the analysis of SSR.

This paper is organized as follows: Section 2 describes the mathematical modeling of STATCOM-ES. Section 3 gives a brief account of bifurcation analysis used, while in Section 4 results obtained from simulation are discussed. The conclusions are drawn in Section 5.

## 2. Modeling of STATCOM-ES

The STATCOM is realized by a 3 level, 12 pulse voltage source converter. This topology also reduces switching losses if the dead angle $\beta$ is varied at fundamental switching frequency [7].In this section, the DQ model of STATCOM-ES is presented.

### 2.1. Modeling of STATCOM-ES in DQ reference frame

STATCOM can be modeled in DQ frame of reference when the switchingfunctions are approximated by their fundamental frequency components by neglecting harmonics.The differential equations governing STATCOM-ES are given by

$$\frac{d_{IsD}}{dt} = \frac{-R_s\omega_B}{X_s} I_{sD} - \omega_0 I_{sQ} + \frac{\omega_B}{X_s}\,[V_{sD} - V_{sD}^i] \quad (1)$$

$$\frac{d_{IsQ}}{dt} = \frac{-R_s\omega_B}{X_s} I_{sQ} + \omega_0 I_{sD} + \frac{\omega_B}{X_s}\,[V_{sQ} - V_{sQ}^i] \quad (2)$$

where $v_s$, $v_s^i$ and $i_s$ are the STATCOM bus voltage, converter output phase voltage and STATCOM current respectively. The converter output voltage is given by

$$V_s^i = \sqrt{{V_{sD}^i}^2 + {V_{sQ}^i}^2} \quad (3)$$

where $V_{sD}^i$ *and* $V_{sQ}^i$ are the D and Q components of converter output voltagewhich are described by

$$V_{sD}^i = K_m * V_{dc} * Sin(\theta + \alpha) \quad (4)$$

$$V_{sQ}^i = K_m * V_{dc} * Cos\,(\theta + \alpha) \quad (5)$$

where$K_m$ = K*cos($\beta$); K = $\frac{2\sqrt{6}}{\pi}$ for a 12 pulse converter, $\alpha$ is the angular difference between the converter voltage and STATCOM bus voltage, $\beta$is the dead angle and '$\theta$ 'is the angle of STATCOM bus voltage described given by

$$\theta = tan^{-1}(\frac{V_{sD}}{V_{sQ}}) \quad (6)$$

Magnitude of STATCOM bus voltage is described by

$$V_s = \sqrt{V_{sD}^2 + V_{sQ}^2} \quad (7)$$

Real and reactive currents injected by STATCOM-ES in D –Q variables are given by

$$I_P = I_{sD}\,Sin\,\theta + I_{sQ}\,Cos\theta \quad (8)$$

$$I_R = -I_{sD}\,cos\,\theta + I_{sQ}\,Sin\theta \quad (9)$$

The real power and reactive power injected or absorbed by the STATCOM-ES are given by

$$P = I_{sD} * V_{sD}^{i} + I_{sQ} * V_{sQ}^{i} \tag{10}$$

$$Q = I_{sQ} * V_{sD}^{i} - I_{sD} * V_{sQ}^{i} \tag{11}$$

*2.2. STATCOM Controller*

For active and reactive power exchange between STATCOM and the network, Type1 controller explained in [3] is required. In this controller, the magnitude and the angle of the output voltage of VSC are varied to control the real and reactive current injected (or absorbed) by the STATCOM. The exchange of real current by VSC is controlled by varying $\alpha$ and the reactive current by varying the magnitude of output voltage of VSC which is a function of dead angle $\beta$. The dynamics of dc bus voltageare neglected. The real current reference is obtained from real power controller. The reactive current reference can either be set constant or controlled to maintain bus voltage constant. The STATCOM controller is as shown in Fig. 1. In this paper, only reactive current control is used.

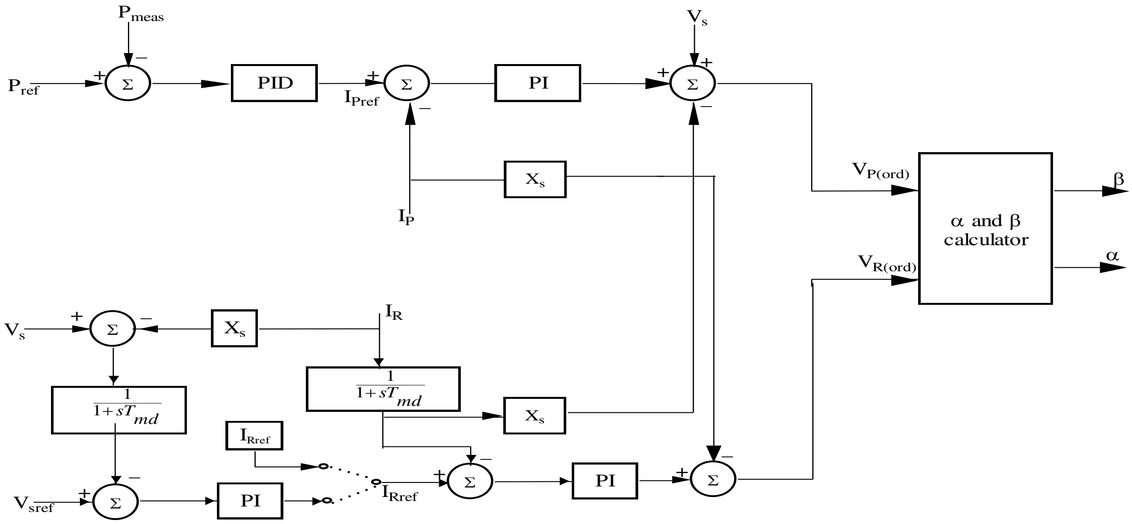



Fig. 1. STATCOM controller

'$\alpha$' and 'β' are calculated by using the following equations

$$\alpha = \tan^{-1}\left(\frac{V_{R(ord)}}{V_{P(ord)}}\right) \tag{12}$$

$$\beta = \mathrm{Cos}^{-1}\left(\frac{\sqrt{V_{P(ord)}^{2}+V_{R(ord)}^{2}}}{k*V_{dc}}\right) \tag{13}$$

*2.3. Test system*

The system considered is adapted from IEEE FBM which incorporates a three level, 12 pulse STATCOM at the electrical center of the transmission line which is as shown in Fig. 2. The line is series compensated by a capacitor. In order to effectively utilize the full rating of STATCOM in both inductive as well as capacitive range, a fixed shunt capacitor is also used at the STATCOM bus. The rating of STATCOM is taken as 150 MVAR. The energy storage device connected to the dc side of STATCOM is capable of exchanging 0.1 pu of real power with the ac system. The synchronous machine is capable of generating 0.9 pu of real power. The mechanical system consists of six masses. The data of the test system are given in appendix.

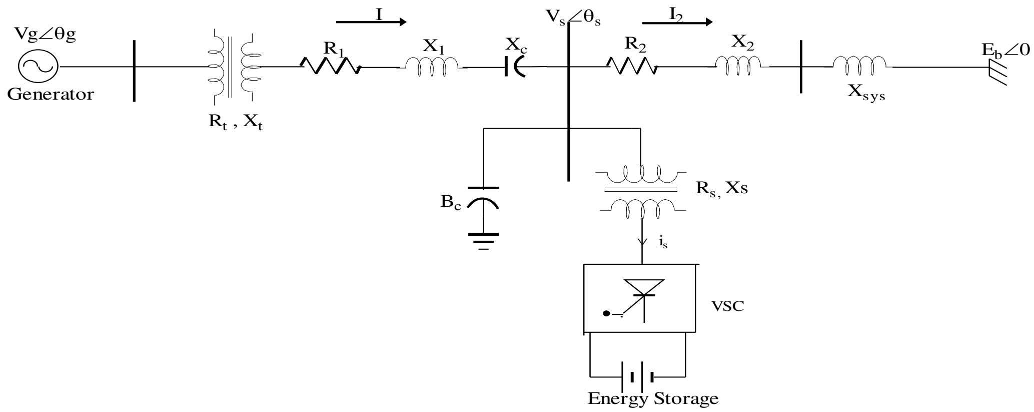



Fig. 2. Modified IEEE FBM with STATCOM-ES

## 3. Bifurcation Analysis of SSR

Electrical power system shows rich nonlinear dynamic phenomena that can be explained by nonlinear analysis of the system. Bifurcation analysis aims at providing explanation for the nonlinear behavior exhibited by power system under the variation of certain parameters. Limit cycle, which is an isolated periodic orbit, is a characteristic behavior of a nonlinear system. Since the presence of Hopf bifurcation indicates the birth or death of limit cycles, the aim of this paper is to detect hopf bifurcation in the system experiencing SSR. Hopf bifurcations are detected by applying Hopf bifurcation theorem given in [11].

Consider a system described by

$$\frac{dx}{dt} = F(x, \mu) \tag{14}$$

When a scalar control parameter $\mu$ is varied, a Hopf bifurcation of an equilibrium solution of the system is said to occur at $\mu = \mu_0$ if the following conditions are satisfied:

1. $F(x_0; \mu) = 0$

2. The system matrix has a pair of purely imaginary eigen values $\pm j\,\omega$ while all of its other eigenvalues have negative real parts at $(x_0; \mu_0)$

3. For $\mu \approx \mu_0$, let the analytical continuation of the pair imaginary eigen values be $\hat{\lambda} \pm j\omega$. Then $\frac{d\lambda}{d\mu} \neq 0$ at $\mu = \mu_0$. This condition implies a transversal or nonzero speed crossing of the imaginary axis and hence is called the transversality condition.

Branches of fixed points and periodic solutions meet at a hopf bifurcation point. Hence, a Hopf bifurcation is classified as a dynamic bifurcation.

## 4. Result Analysis

The test system is modeled in Matlab/Simulink environment. A 2.2 model of machine with static exciter and power system stabilizer is used for the analysis. The mechanical damping is considered. The generator output power Pg is considered to be 0.9pu, with constant mechanical input torque. For the bifurcation analysis, series compensation level $\mu$ is considered as the bifurcation parameter. It is varied from 10% to 90% of line reactance $X_l$.

For the transient simulation, a step change of 10% of mechanical input torque applied at 0.2s and removed at 0.3s is considered. The study is carried out for the following three cases.
Case 1: with STATCOM but no energy storage is connected, Case 2: with STATCOM-ES absorbing real power, and Case 3: with STATCOM-ES injecting real power.

### *4.1. Bifurcation analysis*

Case-1: In this case, STATCOM is connected to the system but there is no energy storage device connected to the dc side of VSC. It is assumed to be operating in capacitive mode and hence injects reactive current into the system. The series compensation levelμ is varied from 10% to 90% of line reactance. The variation of real part of eigen values of torsional modes 1,2,3 and4 are plotted with respect to μ and is shown in Fig. 3(a).Fig. 3(b) shows the variation in rotor angle with μ and Fig. 3(c) shows the existence of limit cycle at $\mu = H_7$.The time series plot of delta when$\mu$=$H_7$ is shown in Fig.3(d). In this case, there are seven hopf bifurcations.

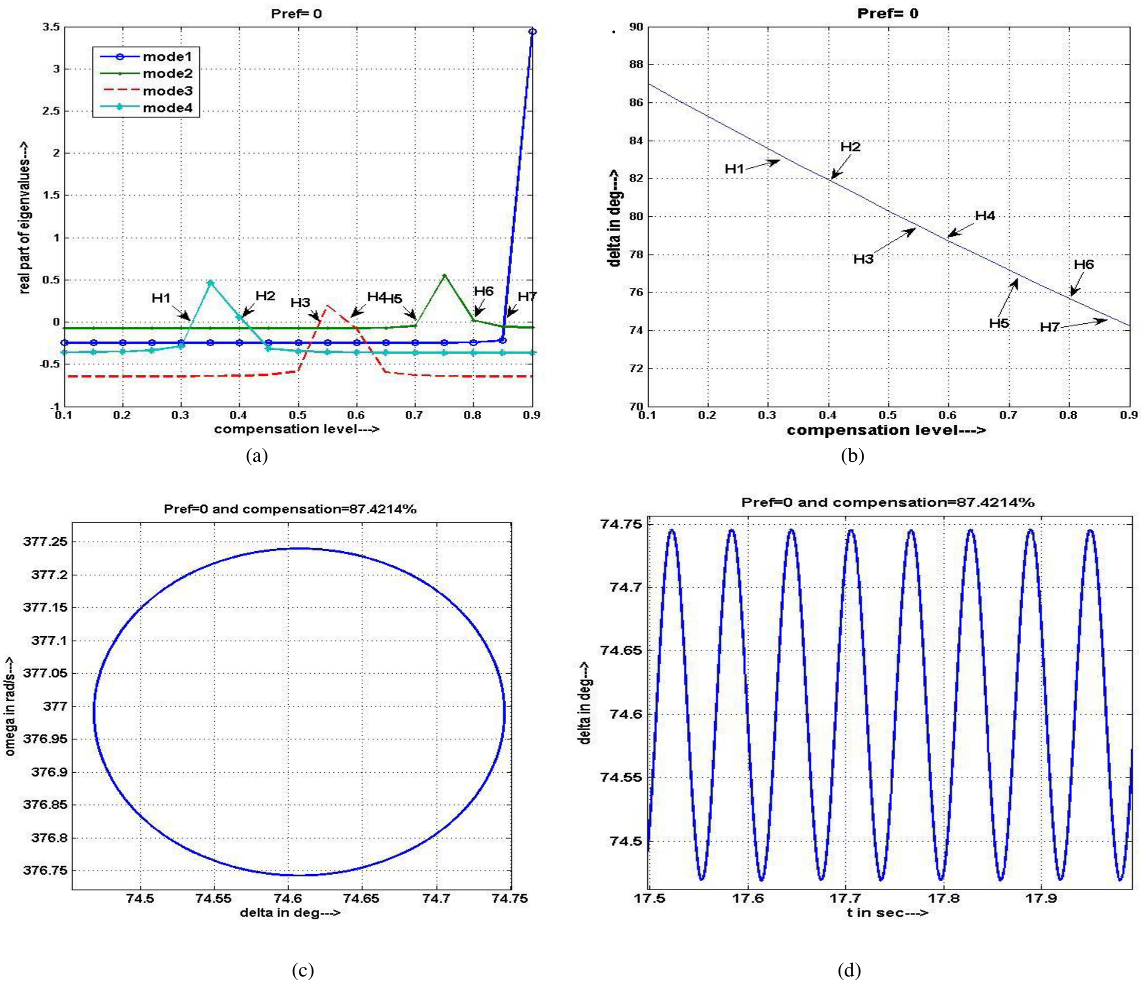



Fig. 3. (a) real part of eigen values of torsional modes with compensation level μ (b) delta vs μ (c) Existence of Limitcycle at $H_7$=0.874214and (d) variation of delta vs time at $H_7$

As the compensation level is increased, the network mode (subsynchronous mode) has an influence on mode 4 and hence the eigen value of mode 4 shifts from left hand side to right hand side of complex plane thus losing stability. The crossing occurs at $\mu = H_1 = 0.33724$. This mode finally regains stability through another Hopf bifurcation which occurs at $\mu = H_2 = 0.4022$. Similarly, when the frequency of electrical mode is very close to the frequency of mode 3, mode 3 loses stability via Hopf bifurcation $\mu = H_3 = 0.54619$ and regains it at $\mu = H_4 = 0.59832$. The torsional mode 2 also loses and regains the stability at $\mu = H_5 = 0.72246$ and $\mu = H_6 = 0.80487$ respectively. The torsional mode 1 loses its stability at $\mu = H_7 = 0.874214$. As $\mu$ is varied further, the power system never regains stability.

Case 2: Here, an energy storage device is connected to the dc side of STATCOM. STATCOM-ES is injecting constant reactive current but absorbing real power Pref=0.1 pu from the AC network. This system again shows seven Hopf bifurcation points at $\mu = H_1 = 0.33753$, $\mu = H_2 = 0.40227$, $\mu = H_3 = 0.546585$, $\mu = H_4 = 0.598677$, $\mu = H_5 = 0.723$, $\mu = H_6 = 0.8048$ and $\mu = H_7 = 0.874574$. At $H_1$, mode 4 loses stability but regains it at $H_2$. At $H_3$, mode 3 loses stability but regains it at $H_4$. Similarly mode2 loses stability via $H_5$ and regains it at $H_6$. When mode 1 interacts with electrical mode, the system becomes unstable via hopf bifurcation $H_7$. The stability of the system is lost because of mode 1.

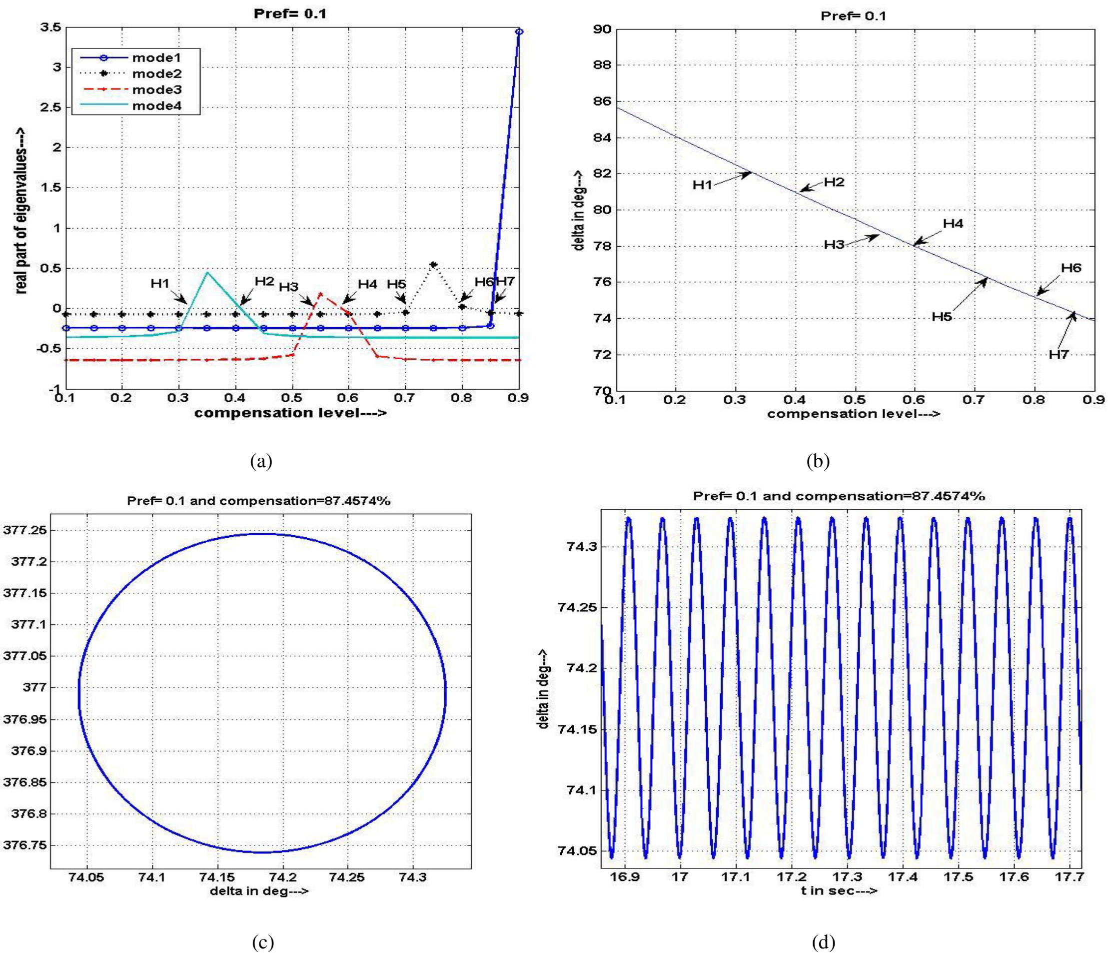
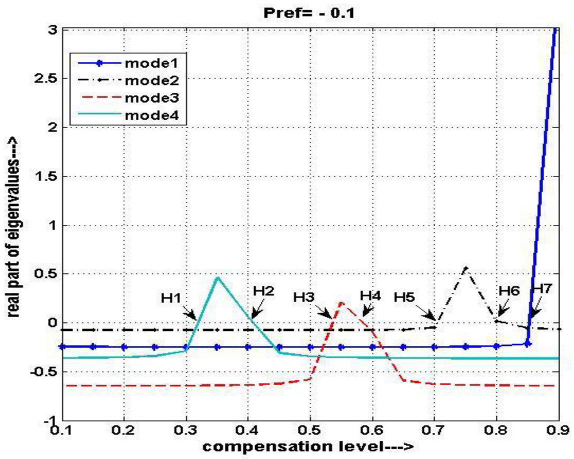


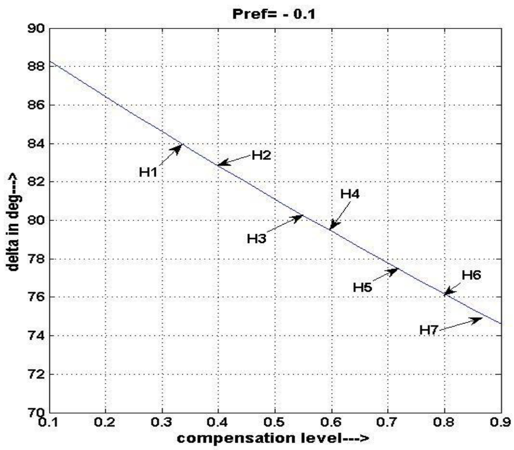

Fig. 4. (a) real part of eigenvalues of torsional modes with compensation level$\mu$ (b) delta with compensation level (c)Existence of Limitcycle at $H_7=0.874574$ (d) variation of delta vs time at $H_7$

Fig. 4(a) shows the variation of real parts of eigen values corresponding to various values of μ. Fig 4(b) shows variation of rotor angle with the bifurcation parameter μ. From Fig 4(c), it can be seen that the phase trajectory of rotor angle with rotor speed exhibits sustained oscillations indicating the presence of limit cycle when μ =0.874574. The time series plot of rotor angle at $H_7$ is shown inFig. 4(d).

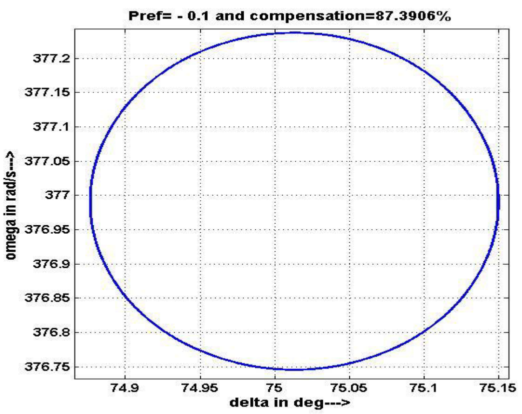
Case-3: In this case, STATCOM-ES is injecting constant reactive current and injecting real power of 0.1 pu to the AC network (Pref= - 0.1 pu). This system again shows seven Hopf bifurcation points at μ =$H_1$=0.33702, μ =$H_2$=0.40217, μ =$H_3$=0.545846, μ =$H_4$=0.598, μ =$H_5$=0.722, μ =$H_6$=0.80496 and μ =$H_7$=0.873906. Mode 4 loses stability via $H_1$but regains it through $H_2$. At $H_3$, mode 3 becomes unstable but regains stability at $H_4$. Similarly mode2 loses stability via $H_5$ and regains it at $H_6$. The interaction of electrical mode with mode 1, results in mode 1 crossing the imaginary axis of complex planeat $H_7$. The stability of the system is lost via mode 1. Fig. 5(a) shows the variation of real parts of eigen values corresponding to various levels of compensation. The variation of rotor angle with the bifurcation parameter μ is shown in Fig. 5(b). Fig. 5(c) and 5(d) show the presence of limit cycle and the variation of delta with respect to time at $H_7$ respectively.



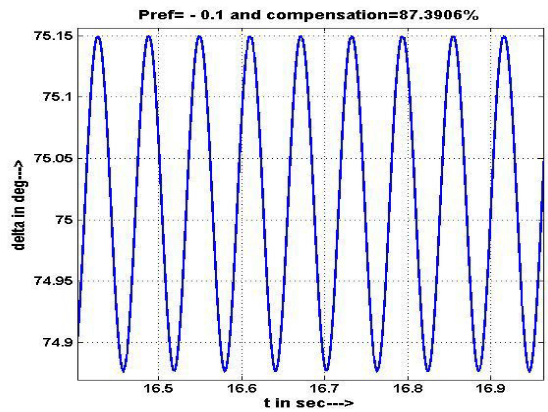

(a)




(b)




(c)




(d)

Fig. 5.(a)real part of eigenvalues of torsional modes with compensation level μ (b) delta with compensation level (c)Existence of Limit cycle at $H_7$=0.873906 (d) variation of delta vs time at $H_7$

Comparing the bifurcation results obtained for case-1 (refer Fig. 3), case-2 (refer Fig. 4) and case-3 (refer Fig. 5), the following observations can be made:-

- The number of Hopf bifurcations remains same and there is existence of limit cycle in all the three cases.
- For all the three cases considered, the system loses stability via mode 1.
- When STATCOM-ES absorbs real power from the power system (case-2), the first hopf bifurcation $H_1$occurs at a slightly higher compensation level as compared to case-1. Hence, there is a marginal increase in stability boundary.
- When STATCOM-ES injects real power into the power system (case-3), the first hopf bifurcation $H_1$occurs at a slightly lower compensation level as compared to case-1. Hence, there is a marginal decrease in stability boundary.
- When STATCOM does not incorporate energy storage (case-1), the stability boundary is slightly less than case-2 but slightly more than case-3.This is because the compensation level at which first hopf bifurcation $H_1$occurs lies between case-2 and case-3.

## 5. Conclusion

The behavior of a compensated transmission line with series capacitor and STATCOM-ES is analyzed using bifurcation technique with compensation level $\boldsymbol{\mu}$ as bifurcation parameter. Transient simulation has been applied to validate the result. The converter is modeled using DQ variables. It is shown that by including STATCOM-ES in the system, the number of Hopf bifurcations remains same and the stability boundary is marginally increased when it absorbs real power. Since there is no significant change in the bifurcation of SSR when the compensation level is varied, it can be inferred that a damping controller needs to be designed for STATCOM-ES to control the bifurcations.

## References

[1] IEEE committee report, Reader's Guide to Subsynchronous Resonance, IEEE Transactions on Power Systems,Vol. 7, No. 1, p.150-158, February 1992.

[2] K..R. Padiyar, Analysis of Subsynchronous Resonance in Power Systems, Kluwer Academic Publishers, Boston: 1999.

[3] K..R. Padiyar, FACTS Controllers in Power Transmission and Distribution, New Age International Publishers: 2008.

[4]A. Arulampalam, J. B. Ekanayake, and N. Jenkins, Application study of a STATCOM with energy storage,IEE Proceedings, vol. 150, No. 3, p. 373–384, 2003.

[5]A. Arsoy, Y. Liu, S. Chen, Z. Yang, M. L. Crow, and P.F. Ribeiro, Dynamic performance of a static synchronous compensator with energy storage, in Proceedings of the IEEE Power Engineering Society Winter Meeting, p. 605–610, February 2001.

[6] MestuBeza, Massimo Bongirno, An Adaptive Power Oscillation Damping Controller by STATCOM with Energy Storage, IEEE Transactions on Power Systems,p.484-493, January 2015.

[7] K..R. Padiyar and Nagesh Prabhu, Design and Performance evaluation of subsynchronous damping controller with STATCOM, IEEE Transactions on Power Delivery,Vol. 21, No. 3, p.1398-1405, July2006.

[8] R. Thirumalaivasan, M. Janaki and Nagesh Prabhu, Damping of SSR using subsynchronous current suppressor with SSSC, IEEE Trans. On Power Syst, 2012.

[9] W. Zhu, R. R. Mohler, R. Spee, W. A. Mittelstadt, and D. Maratukulam, Hopf Bifurcations in a SMIB Power System with SSR, IEEE Proceeding of the 1995 Summer Meeting, p. 531-534, PWRS.

[10] A. H. HarbApplication of Bifurcation Theory to Subsynchronous Resonance in Power Systems, Ph.D Thesis, Virginia Polytechnic Institute and State University, December 1996.

[11]A.H.Nayfeh and BalakumarBalachandran, Applied Nonlinear Dynamics, WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim, 2004.